MATH-436 Introduction to Approximation Theory

Credit Hours: 3-0

Prerequisite: None

Course Objectives: Classical topics like Taylor series and Fourier series deal with approximation of a function by relatively simpler functions. Weierstrass Theorem leads to the fact that every contiuous function can be approximated to arbitrary accuracy by a polynomial of a suitable degree. Fourier series approximates a function f in terms of members of a complete orthonormal set of functions. Smoothness of f is reflected in the fast convergence of the series to

This course leads the student through these topics to the relatively modern concept ofwavelets. This is an important branch of mathematics with technical applications in several areas such as signal processing. This has applications to solving boundary value problems which model problems in diverse areas of science and technology. As a branch of mathematics, the theory leads to approximation in function spaces such as a Hilbert space or a Sobolev space.

Course Contents: Approximation with polynomials; infinite series; best n-term approximation, Fourier analysis, Parseval Theorem; wavelets, application of wavelets, multiresolution analysis, Haar wavelets, Daubechies' wavelets.

Detailed Course Contents: Approximation of a function on an interval; Weierstrass' Theorem, Taylor's Theorem. Infinite series of numbers, estimating the sum of a series, Power series. Series of functions, uniform convergence. Fourier series, Fourier Theorem and approximation.

Fourier series and signal analysis, Generalized Fourier series, Hilbert spaces. Fourier series in complex form, Parseval's Theorem. Regularity and decay of Fourier coefficients. Best n-term approximation, The Fourier transform.

Wavelet systems, the Haar wavelet, Daubechies' wavelets, Wavelets and signal processing, Wavelets and fingerprints, wavelet packets. Multiresolution analysis, The role of Fourier transform.

Learning Outcomes: On successful completion of this course the students will be able to:

- understand the basics of approximation theory,
- apply the wavelet method to solve simple problems,
- use softwares such as Mathematica, Maple, Matlab etc. to implement built-in codes for the solution of problems.

Text Book: Approximation Theory: From Taylor Polynomials to Wavelets, O. Christensenand K.L. Christensen, Birkhauser, Berlin, 2004.

Reference Book: E.W. Cheney, Introduction to Approximation Theory, McGraw-Hill, New York, 1966

Weekly Breakdown		
Week	***Section***	***Topics***
1	1.1- 1.3	Approximation of a function on an interval; Weierstrass' Theorem, Taylor's Theorem.
2	2.1-2.2	Infinite series of numbers, estimating the sum of a series, Power series.
3	20 2.3, 2.5	Series of functions, uniform convergence.
4	28 3.1-3.2	Fourier series, Fourier Theorem and approximation.
5-6	3.3	Fourier series and signal analysis
7	3.4	Generalized Fourier series, Hilbert spaces.
8	3.5-3.6	Fourier series in complex form, Parseval's Theorem.
9	**Mid Semester Exam**	
10	3.7-3.8	Regularity and decay of Fourier coefficients. Best n-term approximation
11	3.9	The Fourier transform
12	4.1	Wavelet systems, the Haar wavelet, Daubechies' wavelets
13	4.2	Wavelets and signal processing
14	4.3-4.4	Wavelets and fingerprints, wavelet packets.
15	5.2-5.3	Multiresolution analysis
16	5.4	The role of Fourier transform.
17		Review
18	**End Semester Exam**	